Coordinate Geometry 1

20 June 2024 18:48

Vector $\rightarrow X = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$ $\quad \mathbb{R}^2 \quad \mathbb{R}^3$

$a = (a_x, a_y)$ $\qquad a = (a_x, a_y, a_z)$

$a = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$ ← column vector $\longrightarrow$ $a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$

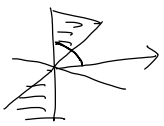
$\in \mathbb{R}^n$

$\tan\theta = \frac{a_y}{a_x}$

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

distance is $\sqrt{a_x^2 + a_y^2}$ ← $\begin{bmatrix} a_x \\ a_y \end{bmatrix}$ $\quad O\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

here it is the euclidean distance



$\begin{bmatrix} a_x \\ a_y \end{bmatrix}$ and $\begin{bmatrix} 2a_x \\ 2a_y \end{bmatrix}$ denotes the same direction

$\bar{x} = \|\bar{x}\|$ → this is the magnitude or norm

↳ unit vector along that direction is $\frac{\bar{x}}{\|\bar{x}\|}$ → some direction $\bar{x}$

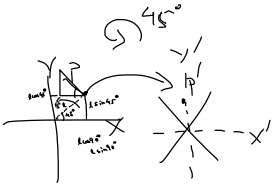
$\left\| \frac{\bar{x}}{\|\bar{x}\|} \right\| = \frac{\|\bar{x}\|}{\|\bar{x}\|\|} = \frac{\|\bar{x}\|}{\|\bar{x}\|} = 1$

$\bar{p}$ • $\rightarrow \bar{q}$ ↳ how to get this direction

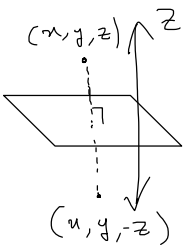
↓

to get the direction of this vector we need to find $\frac{(\bar{q} - \bar{p})}{\|\bar{q} - \bar{p}\|}$ ← some way

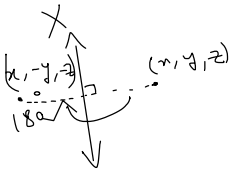
$v = (v_1, v_2, \ldots, v_n)$ $\quad u = (u_1, u_2, \ldots, u_n)$ $\quad \Rightarrow v \pm u = (v_1 \pm u_1, v_2 \pm u_2, \ldots, v_n \pm u_n)$



$P = (1, 2, 3)$ $\xrightarrow{\text{reflected on x-y plane}}$ $Q = (1, 2, -3)$



$R = (1, -2, 3)$ ← rotated by 180° about x-axis



5 units along y

$S = (1, 3, 3)$

Q) (n_1, n_2, n_3) , $n_i \in \{1, 2, \dots, 10\}$

No two are consecutive and $n_1 < n_2 < n_3$

$^{10}C_3 \to n_1 < n_2 < n_3$

Ans:- $n_3 \in \{\cancel{3}, \cancel{4}, 5, \dots, 10\}$

$n_2 \in \{\cancel{2}, 3, \dots, 10\}$

$n_1 \in \{1, 2, \dots, 10\}$

$^{10}C_3 - \underbrace{\text{cases of } n_1, n_2 \text{ cons}}_{A_1} - \underbrace{\text{cases of } n_2 n_3 \text{ cons}}_{A_2} + \underbrace{\text{cases of } n_1 n_2 n_3 \text{ consecutive}}_{A_3}$

$A_1 = n_1 < n_2 < n_3$

$$8 \begin{cases} 1 & 2 \\ 2 & 3 \\ 3 & 4 \\ \vdots & \\ 8 & 9 \end{cases} \leftarrow 10,\ 9,\ 7,\ \dots,\ 3,\ 1$$

$\Rightarrow A_1 = 8 + 7 + 6 + \dots + 1 = 36$

$A_2 = n_1 < n_2 < n_3$

$$1 \to \left.\begin{matrix} 2 & 3 \\ 3 & 4 \\ \vdots & \\ 9 & 10 \end{matrix}\right\} 8$$

$2 \to 7$

$8 \to 1$

$A_2 = 36$

$A_3 = n_1 < n_2 < n_3$

$$A_3 = 8 \begin{cases} 1 & 2 & 3 \\ 2 & 3 & 4 \\ & \vdots & \\ 8 & 9 & 10 \end{cases}$$

$\frac{10 \times 9 \times 8}{6}$ (cancelled to 5, 3 over 6, 2)

$\Rightarrow$ Total cases $= {}^{10}C_3 - 36 - 36 + 8 = 120 - 72 + 8$

$= 128 - 72 = 56$